Mark Scheme (Results)

Summer 2015

Pearson Edexcel International A Level
in Core Mathematics C34 (WMA02/01)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2015
Publications Code IA041208

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 125.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper or ag- answer given
   - $\sqsubset$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through.

After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

   $(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to x = …

   $(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to x = …

2. Formula

   Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

   Solving $x^2+bx+c=0$: $(x\pm\frac{b}{2})^2\pm q\pm c,\quad q\neq 0$, leading to x = …

Method marks for differentiation and integration:

1. Differentiation

   Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

   Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1. (a)	$\left(\frac{dy}{dx}\right) = 8x - 2y\frac{dy}{dx} + 2x\frac{dy}{dx} + 2y = 0$	M1B1A1
	Either Way 1: Sets $\frac{dy}{dx} = 2$ in each term in their differentiated expression	dM1
	$\Rightarrow 8x - 4y + 4x + 2y = 0, \Rightarrow y - 6x = 0$ *	ddM1,A1*
	Or Way 2: Obtains $\frac{dy}{dx} = \left(\frac{8x+2y}{2y-2x}\right)$ (ft their differentiated expression)	dM1
	$\frac{8x+2y}{2y-2x} = 2$, so $y - 6x = 0$*	ddM1,A1*
		(6)
(b)	Put $y = 6x$ or $x = \frac{y}{6}$ into $4x^2 - y^2 + 2xy + 5 = 0$ and obtains $Ay^2 = B$ or $Ax^2 = B$ where A and B are constants	M1
	$x = \pm\frac{1}{2}$ **or** $y = \pm 3$ **or** $\left(\frac{1}{2}, 3\right)$ **or** $\left(-\frac{1}{2}, -3\right)$	A1
	both $\left(\frac{1}{2}, 3\right)$ and $\left(-\frac{1}{2}, -3\right)$ and no extra solutions	A1
		(3)
		(9 marks)

Notes for Question 1

(a) M1 Differentiating $4x^2 - y^2$ with respect to x to obtain $Ax + By\frac{dy}{dx}$

Condone $\frac{dy}{dx} = ...$ at start. May not have lost the +5

B1 Sight of $\frac{d}{dx}(2xy) = 2x\frac{dy}{dx} + 2y$

A1 A fully correct derivative. Accept $8x\,dx - 2y\,dy + 2x\,dy + 2y\,dx = 0$ (needs = 0)

dM1 depends on previous M mark and B mark so has at least two terms in $\frac{dy}{dx}$ and at least two other terms.

Way 1: Sets $\frac{dy}{dx} = 2$ in each term in their differentiated function.

Or Way 2: May see algebra used to give $\frac{dy}{dx} =$ (condone sign slips and slight copying errors but not omission of terms)

ddM1 Dependent upon **both** previous M's. It is for proceeding to obtain an unsimplified **correct equation** in x and y equivalent to those in the scheme e.g. $\frac{-(4x + y)}{x - y} = 2$

A1* cso $y - 6x = 0$ no errors should have been seen - the solutions in the mark scheme would gain full marks (extra lines of working are not required) – accept $y = 6x$

(b)M1 Substitutes $y = 6x$ or $x = \frac{y}{6}$ into the equation of curve C to form an equation in one variable and reaches two term quadratic $Ax^2 = B$ or $Ay^2 = B$ for any values of A or B

$(4x^2 - 36x^2 + 12x^2 + 5 = 0 \Rightarrow)$ $20x^2 = 5$

$\left(4\left(\frac{y}{6}\right)^2 - y^2 + \frac{y^2}{3} + 5 = 0 \Rightarrow\right)$ $\frac{5}{9}y^2 = 5$ but condone slips in the working

A1 Either **one correct pair** of coordinates **or both x or both y values**.

A1 **Both** $\left(\frac{1}{2}, 3\right)$ and $\left(-\frac{1}{2}, -3\right)$ (two answers correct with no incorrect working implies M1A1A1) Any extra solutions (obtained by substituting the variable found first into the quadratic, instead of the linear equation for example) result in A0.

Unsimplifed answers lose the final A mark e.g. $\left(\frac{\sqrt{5}}{\sqrt{20}}, 3\right)$ and $\left(-\frac{\sqrt{5}}{\sqrt{20}}, -3\right)$

Allow (0.5, 3) and (-0.5, -3) – need not be in brackets if pairing is clear but wrong pairing is A0.

$x = \frac{1}{2}, y = 3$ and $x = -\frac{1}{2}, y = -3$ is acceptable but $x = \pm\frac{1}{2}, y = \pm 3$ is not sufficient

Question Number	Scheme	Marks
2(a) **Way 1**	$4(x^2+6) = A(2+x)^2 + B(1-2x)(2+x) + C(1-2x)$	M1
	Let $x=-2 \Rightarrow 40 = 5C \Rightarrow C = 8$	dM1
	Let $x = \frac{1}{2} \Rightarrow 25 = 6.25A \Rightarrow A = 4$ $\quad A=4, C=8$	A1
	Compare constants / terms in *x* or substitute another value of x into identity and conclude that $B=0$ e.g. $24 = 4A+2B+C \Rightarrow B = 0*$	A1* **(4)**
Way 2 **(a)**	$4(x^2+6) = A(2+x)^2 + B(1-2x)(2+x) + C(1-2x)$	M1
	Compare x^2 : so $4 = A-2B$, x: so $0 = 4A-3B-2C$, constants: so $24 = 4A+2B+C$	dM1
	So $A=4, C=8$, and $B=0*$	A1, A1
Way 1 **(b)**	$\frac{4(x^2+6)}{(1-2x)(2+x)^2} = 4(1-2x)^{-1} + 8(2+x)^{-2} = 4(1-2x)^{-1} + 8\times\frac{1}{2^2}\left(1+\frac{x}{2}\right)^{-2}$	B1 ft
	See...$\left(1+(-1)(-2x)+\frac{(-1)(-2)(-2x)^2}{2!}\right)$ or ...$\left(1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{2}\right)^2+\right)$	M1
	...$\left(1+(-1)(-2x)+\frac{(-1)(-2)(-2x)^2}{2!}\right)$ and ...$\left(1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{2}\right)^2+\right)$	A1
	$= 4(1+2x+4x^2+..)+2\left(1-x+\frac{3}{4}x^2\right)$ $\quad = 6+6x+\frac{35}{2}x^2$	dM1A1 **(5)**
Way 2 **(b)**	Or $\frac{4(x^2+6)}{(1-2x)(2+x)^2} = 4(x^2+6)\times(1-2x)^{-1}\times\frac{1}{2^2}\left(1+\frac{x}{2}\right)^{-2}$	B1
	See...$\left(1+(-1)(-2x)+\frac{(-1)(-2)(-2x)^2}{2!}\right)$ or ...$\left(1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{2}\right)^2+\right)$	M1
	...$\left(1+(-1)(-2x)+\frac{(-1)(-2)(-2x)^2}{2!}\right)$ and ...$\left(1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{2}\right)^2+\right)$	A1
	$= 4(x^2+6)(1+2x+4x^2+..)\times\frac{1}{4}\left(1-x+\frac{3}{4}x^2\right) = 6+6x+\frac{35}{2}x^2$	dM1A1 **(5)** **(9 marks)**

Notes for question 2

(a) M1 Uses correct form $4(x^2+6) = A(2+x)^2 + B(1-2x)(2+x) + C(1-2x)$ allow sign errors.

dM1 Uses correct method, either substitution or equating coefficients of terms to find at least one constant (see Way 1 and Way 2 above)

A1 Both $A = 4, C = 8$ (**Correct answers with no working** (cover up rule) imply M1M1A1)

A1* This **needs a method** as the answer is given and **needs to conclude that** $B = 0$.
For method: May compare constants i.e. $24 = 4A + 2B + C$ with $A = 4$ and $C = 8$,
or terms in x i.e. $0 = 4A - 3B - 2C$ with $A = 4$ and $C = 8$,
or term in x^2 : so $4 = A - 2B$ with $A = 4$
or substitute another value for x such as $x = 0$ or $x = 1$ …with $A = 4$ and $C = 8$.

(b) Way 1

B1ft Writes their expression in the form $A(1-2x)^{-1} + C \times \frac{1}{2^2}\left(1+\frac{x}{2}\right)^{-2}$ ft on values of A and C .
There should be no B term.
(This may be awarded for writing or using $A(1-2x)^{-1} + C(2+x)^{-2}$ and writing separately in their solution $C(2+x)^{-2} = C \times \frac{1}{2^2}\left(1+\frac{x}{2}\right)^{-2}$) This could appear in the binomial expansions.

M1 Uses the binomial expansion correctly for one expansion, with power of 2 outside the second bracket ignored. Allow missing brackets for this mark. Ignore constants outside the bracket for this method mark. One completely correct expansion in the bracket

A1 'Both' expansions in the brackets correct and unsimplified. Can be awarded for the two completely correct expansions in the brackets without mention of "A" and /or "C" or adding and with power of 2 outside the second bracket ignored.

dM1 Multiplies out brackets and collects terms. Dependent upon **previous M** mark. Allow sign slips.

A1 $6 + 6x + \frac{35}{2}x^2$. May be written as list or in reverse order. Implies previous M mark.

Way 2

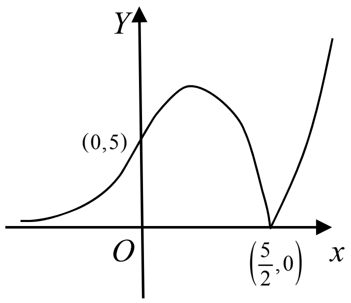
B1 Writes $\frac{4(x^2+6)}{(1-2x)(2+x)^2} = 4(x^2+6) \times (1-2x)^{-1} \times \frac{1}{4}\left(1+\frac{x}{2}\right)^{-2}$. This could be given after the two binomial expansions. (It may be awarded for writing $4(x^2+6) \times (1-2x)^{-1} \times (2+x)^{-2}$ and writing separately $(2+x)^{-2} = \frac{1}{2^2}\left(1+\frac{x}{2}\right)^{-2}$)

M1A1: Follow the scheme and the notes for Way 1

dM1 Must multiply out three brackets for this method

A1 $6 + 6x + \frac{35}{2}x^2$. May be written as list or in reverse order. Implies previous M mark.

N.B. $(2+x)^{-2} = 2^{-2} + (-2)2^{-3}(x) + \frac{(-2)(-3)}{2!}2^{-4}(x)^2 + .. = (\frac{1}{4} - \frac{1}{4}x + \frac{3}{16}x^2)$ is an alternative correct expansion and implies the B1 mark as well as contributing to the M1(one correct expansion) A1 (two correct expansions)

Question Number	Scheme	Marks
3 (a)	$f'(x) = e^x \times 2 + (2x-5)e^x$	M1A1
	$f'(x) = 0 \Rightarrow (2x-3)e^x = 0 \Rightarrow x = \frac{3}{2}$	M1A1
	{Coordinates of $A = \left(\frac{3}{2}, -2e^{\frac{3}{2}}\right)$} obtains $y = -2e^{\frac{3}{2}}$	A1ft
		(5)
(b)	$-2e^{\frac{3}{2}} < k < 0$	M1A1
		(2)
(c)	 Shape including cusp	B1
	$\left(\frac{5}{2}, 0\right)$ only	B1
	$(0,5)$ only	B1
		(3)
		(10 marks)

Notes for Question 3

(a) M1 For applying the product rule correctly to get a form $Ae^{x}+(2x-5)e^{x}$ with A a constant

A1 correct differentiation – need not be simplified - isw

M1 Sets their $f'(x)=0$ and proceeds to $x=..$

A1 Correct answer in any equivalent form. (Ignore extra answers such as ln0 or even 0)

A1ft For finding their correct exact *y* coordinate(ft their *x*) .Allow even if positive. May not be given as coordinates.

Allow $y=-2e^{\frac{3}{2}}$ for this mark

(b) M1 Uses *their* minimum *y* value as lower limit.

Accept $x \geqslant \text{their} -2e^{\frac{3}{2}}$ or $x > \text{their} -2e^{\frac{3}{2}}$ (may use any letter for the M mark)

A1 $-2e^{\frac{3}{2}} < k < 0$ with both inequalities strictly less than – this is cao (so $-2e^{\frac{3}{2}} \le k < 0$ is A0). Need to use *k* this time, not *y* and need exact lower limit.

(c) B1 Correct shape – curve lies only in first two quadrants with maximum in first quadrant. Tends to *x* axis as *x* becomes large and negative . Crosses *y* axis and touches *x* axis with discontinuous gradient –cusp- (not a minimum point). Then gradient becomes steeper as *x* becomes large and Positive. (Give bod if curve looks like straight line here but must not bend back on itself)

B1 Correct *x* coordinate– may be on sketch or in the text. Diagram takes priority over text if there is a contradiction. Need both coordinates if in the text but on the *x* axis the 2.5 is sufficient (even allow (0, 2.5) on *x* axis) **Must be the only crossing point**.

B1 Correct *y* coordinate– may be on sketch or in the text. Need both coordinates if in the text but on the *y* axis the 5 is sufficient (even allow (5,0) on *y* axis). Must be only crossing point.

Question Number	Scheme	Marks
4 (a)	Uses $\mathbf{a}.\mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta \Rightarrow 20 = 5\times 6\cos\theta \Rightarrow \cos\theta = \frac{20}{30} = \left(\frac{2}{3}\right)$	M1A1
		(2)
(b)	Uses $c^2 = a^2 + b^2 - 2ab\cos C \Rightarrow c^2 = 5^2 + 6^2 - 2\times 5\times 6\times\frac{2}{3}$ | $c^2 = a^2 + b^2 - 2\mathbf{a}.\mathbf{b} \Rightarrow c^2 = 5^2 + 6^2 - 2\times 20$	M1
	$\Rightarrow \lvert\overrightarrow{AB}\rvert = \sqrt{21}$	A1
		(2)
(c)	Uses any method (or no method) with their $\cos\theta = \frac{2}{3} \Rightarrow \sin\theta = \left(\frac{\sqrt{5}}{3}\right)$ or gives exact height $= \left(\frac{5\sqrt{5}}{3}\right)$	M1
	Area of triangle $OAB = \frac{1}{2}\times 5\times 6\times\sin(AOB)$	M1
	$\frac{1}{2}\times 5\times 6\times\sin(AOB) = 5\sqrt{5}$ (no evidence of calculator and clear working with surds) (See notes for other methods)	A1*
		(3)
		(7 marks)

Notes for Question 4

(a) M1 Uses $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ to obtain $\cos\theta = ..$ or writes $\theta = \arccos\left(\frac{20}{6\times 5}\right)$

A1 Obtains $\cos\theta = \frac{20}{30}$ *or* $\frac{2}{3}$ then isw (This answer implies M1A1) (isw if they go on to find the angle)

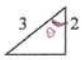
$\cos\theta = \frac{20}{5\times 6}$ earns M1A0

Special case: Uses printed answer to part (c) to find angle then deduces $\cos\theta = \frac{2}{3}$ is M0A0

(b) M1 Uses correct version of the cosine rule with their $\cos\theta = ..$ or $\theta = ..$ to find $|\overrightarrow{AB}|^2 = or\ c^2 =$ (may include non exact angles or cosines)
May be done by splitting into two right angles triangles correctly and using trigonometry and Pythagoras. Method must be completely correct.

A1 For correct **exact** answer only

(c) M1 Uses any method with their **exact** numerical $\cos\theta$ to find **exact** value of $\sin\theta$

Could use $\sin^2\theta + \cos^2\theta = 1$ or Pythagoras' theorem on 

to find $\sin\theta$
Use of angle = 48.19 or any **non exact** work is M0

Just writes down $\sin\theta = \left(\frac{\sqrt{5}}{3}\right)$ should be given the mark bod

May find height of triangle by correct Pythagoras using $5^2 - 25\cos^2\theta = h^2$

M1 Uses a correct method - the formula $\frac{1}{2}5\times 6\sin C$ is most likely but may find height of triangle by trigonometry (or Pythagoras see first M1 mark) and use $\frac{1}{2}b\times h$, with values for b and h (usually 6 and 5sin$C$) For this mark **non exact work may be seen.**

A1 Completely correct **exact** work (without using calculator approximation) and **states answer**
Sight of 0.74535.. for $\sin\theta$ or 3.726.. for height of triangle or the numerical value of the angle (48.2) should be awarded A0

(c)Alternative Method: A neat method is to use

Area $= \frac{1}{2}\sqrt{\{(\mathbf{a}.\mathbf{a})(\mathbf{b}.\mathbf{b}) - (\mathbf{a}.\mathbf{b})^2\}} = \frac{1}{2}\sqrt{\{(25)(36)-(20)^2\}}$ This is M2 then A1 for achieving the printed answer

Alternative method for (c) using **Heron's formula** – can get M1M1A1

M1 Attempts to use $A = \sqrt{s(s-a)(s-b)(s-c)}$ with $s = \frac{a+b+c}{2}$ and $a = 5, b = 6$ and $c =$ their **exact** answer to part (b)

M1 Attempts to simplify their $\sqrt{\left(\frac{11+\sqrt{21}}{2}\right)\left(\frac{11+\sqrt{21}}{2}-5\right)\left(\frac{11+\sqrt{21}}{2}-6\right)\left(\frac{11+\sqrt{21}}{2}-\sqrt{21}\right)}$
by any means (even using a calculator) so this may be inexact

A1 Needs to show $A = \frac{1}{4}\sqrt{(11+\sqrt{21})(1+\sqrt{21})(\sqrt{21}-1)(11-\sqrt{21})} = \frac{1}{4}\sqrt{(121-21)(21-1)}$

$= \frac{1}{4}\sqrt{(2000)} = 5\sqrt{5}$ (Seeing calculator approximation is A0 e.g. 14.3)

Question Number	Scheme	Marks
5.(i)	$\frac{dy}{dx}=\frac{(x+1)-x}{(x+1)^2}=\frac{1}{(x+1)^2}$ or $y=\frac{x}{x+1}=1-\frac{1}{x+1}\Rightarrow\frac{dy}{dx}=\frac{1}{(x+1)^2}$	M1
	(see notes for further methods)	
	$\frac{1}{(x+1)^2}=\frac{1}{4}$ *or* $(x+1)^2=4$ *or* $x^2+2x+1=4$	A1
	$x=1,-3$	M1 A1
		(4)
(ii)	$\int\frac{t+1}{t}\,dt=\int 1+\frac{1}{t}\,dt=t+\ln t\ \ (+c)$ see notes for integration by parts.	M1A1
	$[t+\ln t]_a^{2a}=\ln 7\Rightarrow 2a+\ln 2a-a-\ln a=\ln 7$	
	$a+\ln\left(\frac{2a}{a}\right)=\ln 7\Rightarrow a=\ln\left(\frac{7}{2}\right)$ or $a=\ln 7-\ln 2$	dM1A1
		(4)
		(8 marks)

Notes for Question 5

(i) M1 **Correct** use of quotient, product, implicit differentiation OR chain rules – may not be simplified – accept any correct answer or correct formula quoted followed by slip.

Correct answers are: $-(-1)(x+1)^{-2}$ (chain rule)

$(x+1)^{-1}+(-1)x(x+1)^{-2}$ (product rule)

$y\,(x+1)=x$ gives $(x+1)\frac{dy}{dx}+y=1$ so $\frac{dy}{dx}=\frac{1-y}{x+1}\ or\ \frac{1}{(x+1)^2}$ (implicit differentiation)

A1 $\frac{dy}{dx}=\frac{1}{4}$ and proceeds to one of the listed correct equations in x only.

i.e. $\frac{1}{(x+1)^2}=\frac{1}{4}\ or\ (x+1)^2=4\ or\ x^2+2x+1=4$

M1 Solves their **quadratic** by usual methods (see notes) to obtain two values for $x=..$

A1 Need both answers - two correct answers for x with no working implies the method here. Ignore y values if they are given as well.

(ii) M1 Writes as a sum of 1 and t^{-1} and integrates this sum giving a sum with one correct term e.g. $t+t^{-2}$

May use parts (**see below for two variants on parts**) If parts are used they must be used accurately (as given below). It is not the most efficient method here and usually results in errors and no marks.

A1 Both terms correct (ignore arbitrary constant) e.g. From parts may obtain $t+\ln t+1$

dM1 Uses limits correct way round and sets $\mathrm{I}(2a)-\mathrm{I}(a)=\ln 7$ and **also uses or states** $\ln 2a-\ln a=\ln 2$ leading to $a=..$

This is dependent upon the previous M having been scored

A1 Correct answer. Accept any correct equivalent e.g. $\ln 3.5$.

ln7 – ln2 is A1 but if it is followed by ln5 this is A0

Parts in integration:

Either: $\int\frac{t+1}{t}\,dt=\frac{1}{t}\left(\frac{t^2}{2}+t\right)-\int-\frac{1}{t^2}\left(\frac{t^2}{2}+t\right)dt=\frac{t}{2}+1+\int\frac{1}{2}+\frac{1}{t}dt$ for M1

$=\frac{t}{2}+1+\frac{t}{2}+\ln t$ for A1 then as before

Or: $\int\frac{t+1}{t}\,dt=(t+1)\ln t-\int\ln t\,dt$ for M1

$=(t+1)\ln t-t\ln t+t$ for A1 then as before

Question Number	Scheme	Marks
6(a)	25e or equivalent decimal - Accept awrt 68	B1
		(1)
(b)	$50 = 25\text{e}^{1-10k}$	M1
	Way 1 $\text{e}^{1-10k} = 2$; Way 2 $\text{e}^{-10k} = 2/\text{e}$; Way 3 $\text{e}^{10k} = \text{e}/2$	A1
	Way 1 $\Rightarrow 1-10k = \ln 2$; Way 2 $-10k = \ln(2/\text{e})$; Way 3 $10k = \ln(\text{e}/2)$	M1
	Way 1 $\Rightarrow k = \dfrac{\ln \text{e} - \ln 2}{10}$; Way 2 $k = \dfrac{-\ln\left(\frac{2}{\text{e}}\right)}{10}$; Way 3 No intermediate step needed	
	$\Rightarrow k = \dfrac{\ln\left(\frac{1}{2}\text{e}\right)}{10}$ *	A1*
		(4)
(c)	Uses $m = 20$ and their numerical k so $20 = 25\text{e}^{1-'k't} \Rightarrow \text{e}^{1-'k't} = 0.8$ o.e.	M1
	$\Rightarrow t = \dfrac{1-\ln 0.8}{'k'}$	dM1
	$\Rightarrow t =$ awrt 40 (years)	A1
		(3)
		(8 marks)

Notes for question 6

(a) B1 for 25e or for numerical answer, e.g. 67.957 – allow awrt 68

(b) M1 Uses $t = 10,\ m = 50,$ in $m = 25\mathrm{e}^{1-kt}$ to give $50 = 25\mathrm{e}^{1-10k}$

A1 $\mathrm{e}^{1-10k} = 2$ (way 1) or $\mathrm{e}^{-10k} = 2/\mathrm{e}$ o.e. (way 2) or $\mathrm{e}^{10k} = \mathrm{e}/2$ (way 3) or $\mathrm{e}^{10k-1} = \frac{1}{2}$ (variant on Way 1) to give a correct equation $\mathrm{e}^{\mathrm{f}(k)} = B$. Some solutions will move from one of these options to another by sound algebra – this is acceptable.

M1 Taking logs **correctly** to give f(k) = ln B
i.e. $1 - 10k = \ln 2$ (way 1) or $-10k = \ln(2/\mathrm{e})$ o.e. (way 2) or $10k = \ln(\mathrm{e}/2)$ (way 3) or $10k - 1 = \ln(1/2)$ (way 4*)
(There are a number of correct alternatives but this line should follow directly from the previous one) This must be a correct equation.

A1* cso- Needs both M marks, everything should have been correct and exact. Makes k the subject of the Formula. Needs an intermediate step for ways 1 and 2 but not for way 3.
e.g. $k = \dfrac{\ln \mathrm{e} - \ln 2}{10}$ (Way 1 or Way 4*) or $k = \dfrac{-\ln(2/\mathrm{e})}{10}$ (Way2) or straight to $k = \dfrac{\ln\left(\frac{1}{2}\mathrm{e}\right)}{10}$ (Way 3)

Must conclude with the printed answer $k = \dfrac{\ln\left(\frac{1}{2}\mathrm{e}\right)}{10}$ or $k = \dfrac{\ln\left(\frac{\mathrm{e}}{2}\right)}{10}$ or $k = \dfrac{1}{10}\ln\left(\dfrac{\mathrm{e}}{2}\right)$ o.e.

.

Special Case Taking the mass as 50 + 25e in part (b) should be treated as misread. Can earn M1A0M1A0 and obtains $k = \dfrac{1 - \ln(2 + e)}{10}$

(c) M1 Uses $m = 20$ and their numerical k in $m = 25\mathrm{e}^{1-kt}$ $\Rightarrow \mathrm{e}^{1-'k't} = 0.8$ (NB k = 0.030685…)

NB $\Rightarrow \mathrm{e}^{(1-'k')t} = 0.8$ is M0 (usually $\mathrm{e}^{0.97t} = 0.8$)

dM1 Use of correct work to reach $\Rightarrow t = \dfrac{1 - \ln 0.8}{'k'}$ or equivalent e.g. $\Rightarrow t = \dfrac{\ln\left(\frac{5\mathrm{e}}{4}\right)}{'k'}$

A1 Allow awrt 40 (may see 39.86 or 39.9). (Decimals are acceptable in part (c)) Do not allow -40

Special Case

If the answer 40 appears with no working or after minimal working where no marks have been scored then award M1M0A0 – special case.

If the first M mark in (c) has been awarded and they give the answer 40 with no further working, then award M1M1A1

Question Number	Scheme	Marks
7(a)	$4\tan 2x - 3\cot x \sec^2 x = 0 \Rightarrow 4\times\frac{2t}{1-t^2} - 3\times\frac{1}{t}\times(1+t^2) = 0$	B1M1A1
	So $4\times 2t^2 - 3\times(1+t^2)(1-t^2) = 0$ and $3t^4 + 8t^2 - 3 = 0$ *	A1*
		(4)
(b)	$3t^4 + 8t^2 - 3 = 0 \Rightarrow (3t^2 - 1)(t^2 + 3) = 0$ so $t =$	M1
	$\tan x\ (t) = \pm\frac{1}{\sqrt{3}}$ *or* ± 0.5774	A1
	$x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$	M1A1
		(4)
		(8 marks)

Notes for Question 7

(a) B1 Uses $4\tan 2x = 4\times\frac{2t}{1-t^2}$ or $4\tan 2x = 4\times\frac{2\tan x}{1-\tan^2 x}$. (B0 for $\tan 2x = \frac{\sin 2x}{\cos 2x}$)

M1 Uses **either** $\cot x = \frac{1}{\tan x}$ *or* $\frac{1}{t}$ **or** $\sec^2 x = 1+\tan^2 x$ or $1+t^2$ (quoted correctly)

A1 Uses **both** $\cot x = \frac{1}{\tan x}$ *or* $\frac{1}{t}$ **and** $\sec^2 x = 1+\tan^2 x$ or $1+t^2$ both quoted correctly

[This M1A1 may also arise from the use of $\cot x = \frac{\cos x}{\sin x}$ with $\sec^2 x = \frac{1}{\cos^2 x}$ and with $\cos^2 x + \sin^2 x = 1$ to reach $\frac{1}{t}+t$; so M1 for all three of these identities and A1 for reaching result.] $\cot x = \frac{1}{\tan x}$ may be implied (by tan*x*cot*x* = 1 for example)

A1* As the answer is given, a <u>fully correct intermediate line</u> of working must be seen (Answer may have terms in a different order but must be equivalent correct equation in *t*)

(b) M1 Solves given quadratic in t^2. (may be one slip copying) Accept correct factorisation (shown), formula, completing square. Must then **also square root**. Need to get to *t* = This may be implied by just one answer for *t* or $\tan x$ (from a graphical calculator)

A1 $\tan x\,(t) = \pm\frac{1}{\sqrt{3}}$ (need both plus and minus) or *awrt* ± 0.5774 (ignore further values of $\tan x$ from their quadratic)

N.B. $\tan^2 x = \frac{1}{3}$ is not enough. This is M0A0

M1 For obtaining two answers for *x* from their answers for *t* (must be in different quadrants if following through wrong *t*) (check on your calculator)

A1 All 4 exact and correct and no extra values in the range
If all four answers are correct but in degrees 30, 210, 150, 330 lose final A mark
If the answers are given as decimals 0.524, 3.67, 2.62 and 5.76 lose final A mark.

Question Number	Scheme	Marks
8 (a)	$\frac{d}{dy}(\ln\tan 2y) = \frac{1}{\tan 2y}\times 2\sec^2 2y$	M1A1
	$= \frac{\cos 2y}{\sin 2y}\times\frac{2}{\cos^2 2y} \;=\; = \frac{k}{\sin 2y\cos 2y}$	M1
	$= \frac{4}{2\sin 2y\cos 2y} \;= \frac{4}{\sin 4y}$	A1* cso
		(4)
(b) Way 1	$\frac{dy}{dx} = 2\cos x\sin 4y \Rightarrow \int\frac{dy}{\sin 4y} = \int 2\cos x\,dx$	B1
	$\Rightarrow \frac{1}{4}\ln\tan 2y = 2\sin x(+c)$	M1A1
Finds limits first	Put $x=0, y=\frac{\pi}{6} \Rightarrow \frac{1}{4}\ln\tan 2\frac{\pi}{6} = 2\sin 0 + c \Rightarrow c = ...\left(\frac{1}{4}\ln\sqrt{3}\ or\ \frac{1}{8}\ln 3\right)$	M1
	Takes exponentials so $\tan 2y = e^{8\sin x + c}$	M1
Finds limits after removing lns	$\tan 2y = e^{8\sin x + c}$ (so $\tan 2y = Ae^{8\sin x}$)	M1 (bM3 on epen)
	Put $x=0, y=\frac{\pi}{6}$, so $A =$ or $e^c =$	M1 (bM2 on epen)
	$\tan 2y = \sqrt{3}\ e^{8\sin x}$	A1 **(6)** **(10 marks)**
(b) Way 2	$\frac{dy}{dx} = 2\cos x\sin 4y \Rightarrow \int\frac{dy}{\sin 4y} = \int 2\cos x\,dx$	B1
	$\Rightarrow -\frac{1}{4}\ln(\text{cosec}4y + \cot 4y) = 2\sin x(+c)$	M1A1
	Sub $x=0, y=\frac{\pi}{6}$	
	$\Rightarrow -\frac{1}{4}\ln(\text{cosec}\frac{2\pi}{3} + \cot\frac{2\pi}{3}) = 2\sin 0 + c \Rightarrow c = ...\left(-\frac{1}{4}\ln\frac{1}{\sqrt{3}} \text{ or } \frac{1}{4}\ln\sqrt{3}\right)$	M1
	$-\frac{1}{4}\ln(\frac{1+\cos 4y}{\sin 4y}) = \frac{1}{4}\ln(\tan 2y) = 2\sin x + \frac{1}{4}\ln\sqrt{3}$ so $\tan 2y = \sqrt{3}\ e^{8\sin x}$	M1A1 **(6)**
(b) Way 3	Special case: Differentiates the answer. Marks available B0M1A0M1M1A1	
	$\tan 2y = A\ e^{B\sin x} \to (2)\sec^2 2y\frac{dy}{dx} = AB\cos x e^{B\sin x}$	**M1**
	$\frac{dy}{dx} = \frac{B\cos x\tan 2y\cos^2 2y}{(2)}$	**M1**
	$\frac{dy}{dx} = \frac{B\cos x 2\sin 2y\cos 2y}{(4)} \;=\; \frac{B\cos x\sin 4y}{(4)}$	**M1**
	$B = 8$ and $A = \sqrt{3}$	**A1 4/6**

Notes for question 8

(a) M1 Uses chain rule to obtain $\frac{\mathrm{d}}{\mathrm{d}y}(\ln \tan 2y) = \frac{1}{\tan 2y} \times A\sec^2 2y$ (A can even be 1)

A1 Correct answer (unsimplified)

M1 Uses identity for $\tan 2y = \frac{\sin 2y}{\cos 2y}$ and for $\sec^2 2y = \frac{1}{\cos^2 2y}$ **and reaches** $\frac{k}{\sin 2y \cos 2y}$

or $k \times \frac{1}{\sin 2y} \times \frac{1}{\cos 2y}$

NB Some use long alternative methods using

$\sec^2 2y = 1 + \tan^2 2y = 1 + \frac{\sin^2 2y}{\cos^2 2y} = \frac{\cos^2 2y + \sin^2 2y}{\cos^2 2y} = \frac{1}{\cos^2 2y}$

There needs to be a complete method leading to $\frac{k}{\sin 2y \cos 2y}$ for the M1

A1* cso Writes their expression in terms of $\sin 4y$ using the identity $\sin 4y = 2\sin 2y \cos 2y$ This is a given answer which must be stated and all aspects of the proof must be correct .(Need to multiply top and bottom of fraction by 2 or use other convincing intermediate step).

(b) B1 Separate terms. Accept without the integral as long as integration is implied by subsequent working.

M1 Use inverse of part (a) (integration – the reverse of differentiation) as well as knowing $\int \cos x \mathrm{d}x = \pm \sin x$ to produce $A \ln \tan 2y = B \sin x (+c)$ (or quotes results)

A1 Correct answer, no need for $(+c)$ (Correct answer implies the M1)

M1 Subs $x = 0, y = \frac{\pi}{6}$ into their integrated expression to find c = (must have c for this mark)

M1 Uses correct ln work to find an unsimplified expression for $\tan 2y$ (must have c) (They may remove lns before finding a value for c)

A1 Correct answer and correct solution. Do not accept $A = \tan\left(\frac{\pi}{3}\right)$ need $A = \sqrt{3}$

Way 2

A scheme is given but most will struggle to complete this method. The first 4 marks are readily accessible but showing the answer is difficult. If in doubt, send to review.

B1 Separate terms as in first method

M1 Use standard integral on scheme and obtain $A \ln(\operatorname{cosec} 4y + \cot 4y) = 2\sin x (+c)$

A1 Correct answer, no need for $(+c)$

M1 Subs $x = 0, y = \frac{\pi}{6}$ into their integrated expression to find c (must have c for this mark)

M1 Uses definitions of cosec and cot together with double angle formulae to give $\tan 2y$ (must have c)

A1 Correct answer and conclusion with correct values for A and B

Way 3 (Special Case)

This assumes the answer and differentiates. This is not a complete answer to the question as it merely shows that the given answer satisfies the differential equation. So this is treated as a misread and four of the six marks are available.

B0 Not available as variables are not separated

M1 Implicit differentiation (There may be sign errors or wrong factors of 2)

A0 Not available

M1 Obtains expression in scheme (may be sign errors or wrong factors of 2)

M1 Uses definitions with double angle formulae to give $\frac{B \cos x \sin 4y}{(4)}$

A1 Correct answer and conclusion with correct values for A and B

Question Number	Scheme	Marks
9(a)	A and B are where $y=0$ so $t^3-9t=0 \Rightarrow t(t^2-9)=0 \Rightarrow t=3 \quad (0 \text{ and } -3)$	M1
	When $t=3, \quad x=15$	A1
	$A=(3,0)$	B1
		(3)
Or	**Special case** - uses answer - $t^2+2t=15 \Rightarrow t=3 \quad (-5)$	M1
	When $t=3, \quad y=0$	A1
	$A=(3,0)$	B1
		(3)
(b)	$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{3t^2-9}{2t+2}$	M1A1
	Substitutes $t=3$ into $\frac{dy}{dx}=\frac{3t^2-9}{2t+2} \Rightarrow$ gradient $=\left(\frac{9}{4}\right)$	M1
	Uses their $\left(\frac{9}{4}\right)$ and $(15,0)$ to produce tangent equation $\quad 9x-4y-135=0$ *	M1 A1*
		(5)
(c)	Substitutes $x=t^2+2t,\ y=t^3-9t,$ into $9x-4y-135=0$	
	$\Rightarrow 9(t^2+2t)-4(t^3-9t)-135=0$	M1
	$\Rightarrow 4t^3-9t^2-54t+135=0$	
	$\Rightarrow (t^2-6t+9)(4t+15)=0 \quad$ or $\Rightarrow (t-3)(t-3)(4t+15)=0$	dM1
	$t=-\frac{15}{4}$	A1
	Coordinates of X are $\left(\frac{105}{16},-\frac{1215}{64}\right)$ or $\left(6\frac{9}{16},-18\frac{63}{64}\right)$	ddM1A1cso
	Accept awrt $(6.56,-18.98)$	
		(5)
		(13 marks)

Notes for Question 9

(a) M1 Sets $y = 0$, so $t^3 - 9t = 0 \Rightarrow t = 3$.

A1 Uses $t = 3$, to state the x coordinate of B is 15

B1 States $A = (3, 0)$ (need not see working) must have both coordinates

S.C. M1 Sets $t^2 + 2t = 15 \Rightarrow t = 3$

A1 Uses $t = 3$, to state the y coordinate of B is 0

B1 States $A = (3, 0)$ (need not see working)

(b) M1 Differentiates $x(t)$ and $y(t)$ (allow one error) and calculates $\frac{dy}{dx}$ by using $\frac{dy/dt}{dx/dt}$ -

A1 $\frac{dy}{dx} = \frac{3t^2 - 9}{2t + 2}$

M1 Substitutes 'their' t=3 into their $\frac{dy}{dx}$ to find the gradient of the tangent at B

M1 Uses (15,0) and their non-zero numerical gradient to find an equation of the tangent.

A1* Achieves given answer cso $\Rightarrow 9x - 4y - 135 = 0$

(c) M1 Substitutes $x = t^2 + 2t$, $y = t^3 - 9t$ into $9x - 4y - 135 = 0$ correctly to form a cubic equation in just t. (Need a correct equation)

N.B. Any attempts to work in just x or in just y are unlikely to achieve a cubic. If there seems to be any success send to review. Equations in a mixture of x, y and t score M0

dM1 Attempts to solve a cubic polynomial = 0. Division by $(t - 3)$ or by $(t - 3)^2$

Use of a graphical calculator is acceptable. Just seeing $(4t + 15)$ is enough but any errors e.g. extra solutions or errors factorising) are penalised by the loss of the final A mark. May use other variables than t for this mark, when trying to factorise.

A1 $t = -\frac{15}{4}$ (This answer with no working implies previous M mark if cubic has been seen)

ddM1 Uses their value of t to find both the x and y co-ordinates. It is dependent upon both the previous M's having been scored

A1cso Coordinates are $\left(\frac{105}{16}, -\frac{1215}{64}\right)$ Accept awrt $(6.56, -18.98)$. Allow for two sets of coordinates if (15,0) is not rejected. But lose this mark if the correct point is rejected in favour of (15,0). Lose this mark for errors factorising cubic earlier, or for extra wrong values of t found earlier. Allow for $x = \frac{105}{16}$, $y = -\frac{1215}{64}$

Question Number	Scheme	Marks
10.	$A=\pi x^2 \Rightarrow \frac{dA}{dx}=2\pi x$ (**condone use of *r* throughout instead of *x***)	B1
	Uses $\frac{dA}{dt}=\frac{dA}{dx}\times\frac{dx}{dt} \Rightarrow \frac{\pi}{20}=2\pi x\times\frac{dx}{dt} \Rightarrow \frac{dx}{dt}=\frac{1}{40x}=\left(\frac{1}{80}\right)$	M1 A1
	$V=6\pi x^3 \Rightarrow \frac{dV}{dx}=18\pi x^2$	B1
	Uses $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt} \Rightarrow \frac{dV}{dt}=18\pi 2^2\times\frac{1}{80}=\frac{9}{10}\pi$	dM1 A1
		(6 marks)
Way 2	$V=6\pi x^3, A=\pi x^2 \Rightarrow V=6\pi\left(\frac{A}{\pi}\right)^{\frac{3}{2}}$	M1
	$\frac{dV}{dA}=9\left(\frac{A}{\pi}\right)^{\frac{1}{2}}$	B1 B1
	Uses $\frac{dV}{dt}=\frac{dV}{dA}\times\frac{dA}{dt} \Rightarrow \frac{dV}{dt}=9\left(\frac{4\pi}{\pi}\right)^{\frac{1}{2}}\times\frac{\pi}{20} \Rightarrow \left.\frac{dV}{dt}\right\|_{A=4\pi}=\frac{9}{10}\pi$	dM1A1 A1 **(6 marks)**
Way 3	$A=\pi x^2 \Rightarrow \frac{dA}{dx}=2\pi x$	First B1
	$V=6\pi x^3 \Rightarrow \frac{dV}{dx}=18\pi x^2$	Second B1
	Uses $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dA}\times\frac{dA}{dt} \Rightarrow \frac{dV}{dt}=18\pi x^2\times\frac{1}{2\pi x}\times\frac{\pi}{20}$	M1A1
	Put $x=2$ to give $\frac{9}{10}\pi$	dM1A1 **(6 marks)**
	Misunderstands **area** as **Total Surface Area** $A=14\pi x^2 \Rightarrow \frac{dA}{dx}=28\pi x$	B0
	Uses $\frac{dA}{dt}=\frac{dA}{dx}\times\frac{dx}{dt} \Rightarrow \frac{\pi}{20}=28\pi x\times\frac{dx}{dt} \Rightarrow \frac{dx}{dt}=\frac{1}{560x}=\left(\frac{1}{1120}\right)$	M1A1
	$V=6\pi x^3 \Rightarrow \frac{dV}{dx}=18\pi x^2$	B1
	$\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt} \Rightarrow \frac{dV}{dt}=18\pi 2^2\times\frac{1}{1120}=\frac{9}{140}\pi$	dM1A0
	Misunderstands **area** as **Curved Surface Area** $A=12\pi x^2 \Rightarrow \frac{dA}{dx}=24\pi x$	B0
	Similar scheme to above with $\frac{\pi}{20}=24\pi x\times\frac{dx}{dt} \Rightarrow \frac{dx}{dt}=\frac{1}{480x}=\left(\frac{1}{960}\right)$	M1A1
	$V=6\pi x^3 \Rightarrow \frac{dV}{dx}=18\pi x^2$	B1
	$\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt} \Rightarrow \frac{dV}{dt}=18\pi 2^2\times\frac{1}{960}=\frac{9}{120}\pi$ $\left(or\ \frac{3}{40}\pi\right)$	dM1A0

Notes for Question 10

Must use calculus.

Way 1 B1 Correct statement $\frac{dA}{dx} = 2\pi x$

M1 Uses **correct** chain rule $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$ or equivalent e.g. $\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx}$

with $\frac{dA}{dt} = \frac{\pi}{20}$ and their $\frac{dA}{dx}$ to calculate $\frac{dx}{dt}$. NB If they correctly state the chain rule $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$ then make an algebraic error they may be awarded this method mark

A1 Obtain correct expression for $\frac{dx}{dt}$ e.g. $= \frac{\pi/20}{2\pi x}$ then isw (award this mark in the two misread cases described where correct Curved Surface Area or Total Surface Area are used correctly)

B1 Correct statement $\frac{dV}{dx} = 18\pi x^2$

dM1 Uses $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$ with their $\frac{dx}{dt}$ and $\frac{dV}{dx}$ to calculate $\frac{dV}{dt}$, with $x = 2$ substituted.

NB $\frac{dV}{dx}$ should be in terms of one variable x (so of the form kx^2 and not kxh)

A1 $\frac{9}{10}\pi$ or 0.9π or $\frac{18}{20}\pi$ ……or k =0.9 etc

Way 2 M1 Writes V in terms of A. Accept $V = ..A^{\frac{3}{2}}$ (first M on epen) This indicates way 2.

B2 $\frac{dV}{dA} = 9\left(\frac{A}{\pi}\right)^{\frac{1}{2}}$ (both Bs on epen)

dM1 This is dependent on the first method mark where $V = ..A^{\frac{3}{2}}$.

Uses $\frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt}$ with their $\frac{dV}{dA}$ and $\frac{dA}{dt} = \frac{\pi}{20}$ to find $\frac{dV}{dt}$ (second M on epen)

NB $\frac{dV}{dA}$ should be in terms of one variable A (so of the form $kA^{\frac{1}{2}}$)

A2 $\frac{9}{10}\pi$ or 0.9π or $\frac{18}{20}\pi$ ……or k =0.9 etc (both As on epen)

Way 3 Similar to Way 1 but does calculation in one stage, not two – see scheme above

Condone use of r throughout, but if there is a mixture of r, x and h, then final accuracy mark may be with- held if full marks would have been gained. (Send to review if in doubt)

NB: Misreads/ misunderstandings of cross section area- there are two examples in the scheme above- another possible is an open cylinder where area = $13\pi x^2 \Rightarrow \frac{dA}{dx} = 26\pi x$ –

then $\frac{\pi}{20} = 26\pi x \times \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{1}{520x} = \left(\frac{1}{1040}\right)$ and then $V = 6\pi x^3 \Rightarrow \frac{dV}{dx} = 18\pi x^2$ which gives $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} \Rightarrow \frac{dV}{dt} = 18\pi 2^2 \times \frac{1}{1040} = \frac{9}{130}\pi$ These may be also combined with Way 3

Question Number	Scheme	Marks
11(a)	$(R=\sqrt{1.5^2+1.2^2})=$ awrt 1.921 - accept e.g. $\sqrt{3.69}$ or $\frac{3\sqrt{41}}{10}$	B1
	$\tan\alpha=\frac{1.2}{1.5}\Rightarrow\alpha=0.675$ or 0.215π	M1A1
		(3)
(b)	$H=3+1.921\sin\left(\frac{\pi t}{6}-0.675\right)$	
	$H_{\min}=3-'1.921'=\text{awrt}1.08$	M1A1
	$\left(\frac{\pi t}{6}-"0.675"\right)=\frac{3\pi}{2}\Rightarrow t=10.29$	M1A1
		(4)
(c)	$4=3+1.921\sin\left(\frac{\pi t}{6}-0.675\right)\Rightarrow\sin\left(\frac{\pi t}{6}-0.675\right)=\frac{1}{1.921}$	M1
	$\frac{\pi t}{6}-0.675=0.548\Rightarrow t=$ awrt 2.33 or 2.34	dM1A1
	$\frac{\pi t}{6}-0.675=\pi-0.548=2.594\Rightarrow t=$ awrt 6.24 or 6.25	ddM1A1
	Times are 2:20pm and 6:15pm or 6.14pm (14:20 and 18:15 or 18:14) – allow 2 hours 20minutes and 6 hours 15 or 14minutes or 140 minutes and 375 or 374 minutes	A1
	Extra values in the range – lose final A mark.	**(6)**
		(13 marks)

Notes for Question 11

(a) B1 $R =$ awrt 1.921 (3dp) – allow any equivalent e.g. $\sqrt{3.69}$ or $\frac{3\sqrt{41}}{10}$

M1 $\tan\alpha = \pm\frac{1.2}{1.5}$ or $\tan\alpha = \pm\frac{1.5}{1.2}$

A1 $\alpha =$ awrt 0.675 (3dp) also allow 0.215π (must be in radians)

(b) M1 States or attempts to calculate $3 - R$ with their value of R

A1 $H_{\min} =$ awrt 1.08. Or 3 - $\frac{3\sqrt{41}}{10}$ o.e.Accept this for both marks as long as no incorrect working is seen.

M1 Attempts $\left(\frac{\pi t}{6} - '\alpha'\right) = \frac{3\pi}{2} \Rightarrow t = ...$

(Putting equal to $-\frac{\pi}{2}$ is M1A0 (outside range) Putting equal to $\frac{\pi}{2}$ is M0A0 (wrong))

(Allow method mark for using -90 or 270 degrees (not 90 degrees), if alpha was in degrees earlier)

A1 $t =$ awrt 10.29 (2dp). Accept this for both marks as long as no incorrect working is seen.

(c) M1 $\sin\left(\frac{\pi t}{6} \pm '\alpha'\right) = \frac{4-3}{R}$, where $\left|\frac{4-3}{R}\right| < 1$ (allow for degrees)

dM1 Dependent upon the previous M, using the correct order to find **one value** of t (allow consistent degrees)

NB: sin (1/1.921) = 0.497 Seeing 0.497 is indication that sin instead of arcsin has been used. This indicates the wrong method and so M0.

A1 Accept either awrt 2.33 or 2.34 **or** awrt 6.24 or 6.25 do not need units – ignore wrong units e.g. minutes and seconds for this mark. 2 hours 20 minutes is correct here. Note that 2 minutes 20 seconds could get this mark but would lose the final A1.

ddM1 Dependent upon the previous M, using the correct order to find a **second value** of t

A1 Accept awrt 2.33 or 2.34 **and** awrt 6.24 or 6.25- ignore wrong units. So 6 minutes 14 seconds could get this mark but would lose the next.

A1 Times are 2:20pm and 6:15pm (or 6.14pm) (14:20 and 18:15 (or 18.14)) (Need both times) – allow 2 hours 20minutes and 6 hours 15 or 14minutes or 140 minutes and 375 or 374 minutes. Extra values in the range – lose final A mark

Allow method marks for degrees, and accuracy marks if they converted to

$\sin(30t \pm '38.65') = \frac{4-3}{R}$, where $\left|\frac{4-3}{R}\right| < 1$ and continued to correct answers. Using

$\sin\left(\frac{\pi t}{6} \pm '38.65'\right) = \frac{4-3}{R}$, where $\left|\frac{4-3}{R}\right| < 1$ will lose the accuracy marks.

Question Number	Scheme	Marks
12(i)	$(\overrightarrow{OP}) = \begin{pmatrix} -5+2\lambda \\ 1-3\lambda \\ 6+1\lambda \end{pmatrix}$ or coordinates of P are $(-5+2\lambda, 1-3\lambda, 6+\lambda)$	B1
	$\overrightarrow{OP} \cdot \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} -5+2\lambda \\ 1-3\lambda \\ 6+1\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = 0$	
	$\Rightarrow 2(-5+2\lambda) - 3(1-3\lambda) + 1(6+1\lambda) = 0 \Rightarrow 14\lambda = 7 \Rightarrow \lambda =$	M1
	$\lambda = \frac{1}{2}$	A1
	Substitute their $\lambda = \frac{1}{2}$ into their $\overrightarrow{OP} = \begin{pmatrix} -5+2\lambda \\ 1-3\lambda \\ 6+1\lambda \end{pmatrix}$	dM1
	P has coordinates $(-4, -0.5, 6.5)$	A1 (5)
(ii)	Way 1 $\overrightarrow{OA} = k \times \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix}$ or the coordinates of A are $(5k, -3k, 4k)$	M1
	$k\sqrt{5^2 + (-3)^2 + 4^2} = \sqrt{2}$ or $k^2\left(5^2 + (-3)^2 + 4^2\right) = 2$ o.e.	A1
	Finds at least one value for k and substitutes into $\overrightarrow{OA}$ to give position	dM1
	As $k = (\pm)\frac{1}{5}$, A has possible positions $\begin{pmatrix} 1 \\ -\frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$ or $\begin{pmatrix} -1 \\ \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$ (any notation)	A1
	As $k = (\pm)\frac{1}{5}$, A has possible positions $\begin{pmatrix} 1 \\ -\frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$ **and** $\begin{pmatrix} -1 \\ \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$ (any notation)	A1 **(5)**
	Way 2 $\overrightarrow{OA} = \begin{pmatrix} x \\ -\frac{3}{5}x \\ \frac{4}{5}x \end{pmatrix}$ or the coordinates of A are $(x, -\frac{3}{5}x, \frac{4}{5}x)$	M1
	$x\sqrt{1^2 + (-\frac{3}{5})^2 + \frac{4}{5}^2} = \sqrt{2}$ or equivalent (see notes for variations)	A1
	Finds x and substitutes into $\overrightarrow{OA}$ to give position	dM1
	As $x = (\pm)1$ so A has coordinates $\left(1, -\frac{3}{5}, \frac{4}{5}\right)$ **or** $\left(-1, \frac{3}{5}, -\frac{4}{5}\right)$	A1
	As $x = (\pm)1$ so A has coordinates $\left(1, -\frac{3}{5}, \frac{4}{5}\right)$ **and** $\left(-1, \frac{3}{5}, -\frac{4}{5}\right)$	A1 **(5)**
	Way 3 (This is a common approach: see next page)	

Question Number	Scheme	Marks
	Way 3	
	Writes $\frac{5}{x} = -\frac{3}{y} = \frac{4}{z}$ This is 2 equations, 3 unknowns	M1
	Writes $\left(x^2+y^2+z^2\right)=2$ This is the third equation in 3 unknowns	A1
	Eliminates two of the variables to obtain either x, y, or z and uses it to find the other values	dM1
	A has coordinates $\left(1, -\frac{3}{5}, \frac{4}{5}\right)$ **or** $\left(-1, \frac{3}{5}, -\frac{4}{5}\right)$	A1
	A has coordinates $\left(1, -\frac{3}{5}, \frac{4}{5}\right)$ **and** $\left(-1, \frac{3}{5}, -\frac{4}{5}\right)$	A1 **(5)**
(ii)	Way 4 (minimal working): States $\frac{\sqrt{2}}{\sqrt{50}}$ or $\frac{\sqrt{50}}{\sqrt{2}}$	M1A1
	Deduces position vectors or coordinates $\mathrm{i}-\frac{3}{5}\mathrm{j}+\frac{4}{5}\mathrm{k}$ **or** $-\mathrm{i}+\frac{3}{5}\mathrm{j}-\frac{4}{5}\mathrm{k}$	M1A1
	$\mathrm{i}-\frac{3}{5}\mathrm{j}+\frac{4}{5}\mathrm{k}$ **and** $-\mathrm{i}+\frac{3}{5}\mathrm{j}-\frac{4}{5}\mathrm{k}$	A1 **(5)** **(10 marks)**

Notes for Question 12

(i) B1 $\overrightarrow{OP} = \begin{pmatrix} -5+2\lambda \\ 1-3\lambda \\ 6+1\lambda \end{pmatrix}$ or coordinates of P are $(-5+2\lambda, 1-3\lambda, 6+\lambda)$. This may be implied by its use in the scalar product.

M1 Attempts to use (may make slip copying) their $\overrightarrow{OP}.\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = 0$ obtaining an equation in λ and solving to give $\lambda =$

They may use any multiple of 2**i** – 3**j** + **k** (including λ (2**i** – 3**j** + **k**) **or –**(2**i** – 3**j** + **k**) etc**…**)

A1 $\lambda = \frac{1}{2}$

dM1 Substitutes their λ into their $\overrightarrow{OP}$. Dependent upon the previous M

A1 P has coordinates $(-4, -0.5, 6.5)$. Accept it written in coordinate or in a vector form – either as column vector or as - 4**i** - 0.5 **j** +6.5**k. Accept** $\overrightarrow{OP} = \begin{pmatrix} -4 \\ -\frac{1}{2} \\ \frac{13}{2} \end{pmatrix}$ for example.

(ii) Way 1

M1 Writing $\overrightarrow{OA} = k \times \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix}$ or the coordinates of A as $(5k, -3k, 4k)$.

This mark may be implied by correct coordinates later.

A1 Correct equation in k (one variable) using 3D Pythagoras' theorem dM1 (Dependent on first M mark) Solves their equation to give at least one value for k (their one variable, even allow use of μ here) and substitutes to find at least one possible position.

A1 $\mathbf{a} = \begin{pmatrix} 1 \\ -\frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$ OR $\begin{pmatrix} -1 \\ \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$. Allow coordinates $\left(1, -\frac{3}{5}, \frac{4}{5}\right)$ **or** $\left(-1, \frac{3}{5}, -\frac{4}{5}\right)$

A1 $\mathbf{a} = \begin{pmatrix} 1 \\ -\frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$ AND $\begin{pmatrix} -1 \\ \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$. Allow coordinates $\left(1, -\frac{3}{5}, \frac{4}{5}\right)$ **and** $\left(-1, \frac{3}{5}, -\frac{4}{5}\right)$

If there are surds in the answer but it is otherwise correct then lose the final A1

Way 2:

This is described in the scheme where x is the single variable used but could be used with $(-\frac{5}{3}y, y, -\frac{4}{3}y)$ where y is $(\pm)\frac{3}{5}$, or with $(\frac{5}{4}z, -\frac{3}{4}z, z)$ where z is $(\pm)\frac{4}{5}$ in a similar way.

Way 3: The successful approach is described above in the scheme.

Solving the equations may be lengthy, they may eliminate their variables one at a time and they may make errors.

M1: This is for these correct equations in the scheme (Two equations – three unknowns)

A1: A correct further equation (third equation – three unknowns)

M1: Solves to obtain the three variables

A1, A1 as before

(A common variation)

Some write $\left(x^2 + y^2 + z^2\right) = 2$ together with a scalar product giving $5x - 3y + 4z = \sqrt{2}\sqrt{50}$

This is two equations in three unknowns and usually stops there. This is marked M0A0M0A0A0 unless the candidate produces a third equation and makes progress towards the answer.

Way 4: (Special Case) M1A1: Writes down $\frac{\sqrt{2}}{\sqrt{50}}$ with little or no working (or $\frac{1}{5}$ or even 5)

dM1A1A1: Uses their correct fraction to find the coordinates (position vectors) – as before

Question Number	Scheme	Marks
13 (a)	awrt 0.3799 – may be seen in the table	B1 **(1)**
(b)	Area = $\frac{1}{2}\times\left(\frac{e^2-e}{2}\right)(1+2\times'0.3799'+0)$ [The +0 is not required]	B1M1
	= awrt 2.055	A1 **(3)**
(c) Way 1	$\int(\ln x)^2\,dx=\int 1\times(\ln x)^2\,dx=x(\ln x)^2-\int x\times\frac{2\ln x}{x}\,dx$	M1
	$=x(\ln x)^2-\int 2\ln x\,dx$	A1
	$=x(\ln x)^2-2x\ln x+\int 2\,dx$	dM1
	$=x(\ln x)^2-2x\ln x+2x(+c)$	A1* (4)
(c) Way 2	$\int(\ln x)^2\,dx=\int(\ln x)\times(\ln x)\,dx=\ln x(x\ln x-x)-\int\frac{1}{x}\times(x\ln x-x)\,dx$	M1
	$=\ln x(x\ln x-x)-\int \ln x-1\,dx$	A1
	$=\ln x(x\ln x-x)-(x\ln x-x-x)$	dM1
	$=x(\ln x)^2-2x\ln x+2x(+c)$	A1* (4)
(c) Way3	Use $u=\ln x$ substitution to get to	
	$\int u^2e^u\,du=u^2e^u-\int 2ue^u\,du$	M1
	$=u^2e^u-\left[2ue^u-\int 2e^u\,du\right]$	A1
	$=u^2e^u-2ue^u+2e^u+k$	
	$=x(\ln x)^2-2x\ln x+2x(+c)$	dM1A1 (4)
(d)	Volume = $\int \pi y^2\,dx=\int\pi(2-\ln x)^2\,dx$	B1
	$\int(2-\ln x)^2\,dx=\int 4-4\ln x+(\ln x)^2\,dx$	M1
	Correct integration of at least two of their three terms (see notes)	M1
	$=4x-4(x\ln x-x)+x(\ln x)^2-2x\ln x+2x\ (+c)$	A1
	Volume $=\pi\left[4x-4(x\ln x-x)+x(\ln x)^2-2x\ln x+2x\right]_{e}^{e^2}$	ddM1
	$=2\pi e^2-5\pi e$	
	$=\pi e(2e-5)$	A1
		(6) **(14 marks)**

Notes for Question 13

(a) B1 awrt 0.3799 – should be in the table or given as answer – not just appear in trapezium rule

(b) B1 For the strip width of $\frac{e^2-e}{2}$ or correct equivalent e.g. $\frac{e^2+e}{2}-e$, or $e^2-\frac{e^2+e}{2}$

Also accept awrt 2.34 o.e.

This may be stated as h = the values above , or may be used correctly in the rule so $\frac{e^2-e}{4}$, etc or 1.17 may be seen.

M1 For **correct application** of the trapezium rule – requires correct ft bracket $....\times(1+2\times\text{their answer to}(a)+0)$

A1 awrt 2.055

(c) **Must use integration by parts or 0/4. Differentiating the answer is not acceptable**.

Way 1 M1 One correct application of integration by parts the correct way around.

Accept $\int(\ln x)^2\,dx = x(\ln x)^2 - \int x\times\frac{A\ln x}{x}dx$

A1 $x(\ln x)^2 - \int 2\ln x dx$

dM1 A second application of integration by parts the correct way around

Accept $= x(\ln x)^2 - Ax\ln x \pm \int B dx$

Accept the answer to the $\int \ln x dx$ part just written down as $x\ln x - x$

A1* Correct solution only $= x(\ln x)^2 - 2x\ln x + 2x(+c)$ with or without 'c'

Way 2 M1 One correct application of integration by parts the correct way around.

Accept $\int(\ln x)^2\,dx = \int(\ln x)\times(\ln x)dx = \ln x(x\ln x - x) - \int\frac{1}{x}\times(x\ln x - x)dx$

A1 $\ln x(x\ln x - x) - \int(\ln x - 1)dx$

dM1 A second application of integration by parts the correct way around

Accept $= \ln x(x\ln x - x) - (x\ln x - x - x)$

Accept the answer to the $\int \ln x dx$ part just written down as $x\ln x - x$ and substituted into their expression

A1* Correct solution only $= x(\ln x)^2 - 2x\ln x + 2x(+c)$ with or without 'c'

Way 3 M1 Uses substitution and performs one correct application of integration by parts the correct way around.

A1 $= u^2e^u - \left[2ue^u - \int 2e^u du\right]$ after second integration by parts

dM1 Final integration and returns to x

A1* Correct solution only $= x(\ln x)^2 - 2x\ln x + 2x(+c)$ with or without 'c'

(d) B1 Volume = $\int \pi y^2 dx = \int \pi(2-\ln x)^2\,dx$ (needs π and integral symbol) but not limits and can condone missing dx

M1 Multiplies out $(2-\ln x)^2$ to $\lambda+\mu\ln x+\nu(\ln x)^2$ where $\lambda,\ \mu$ and ν are non zero positive or negative constants.

M1 Needs attempt to multiply out to at least $\lambda+\nu(\ln x)^2$ where λ and ν are non zero positive or negative constants.and attempt to integrate.

Look for $\lambda x\pm\mu(x\ln x-x)\pm\nu\left(x(\ln x)^2-2x\ln x+2x\right)$ with **two of the three** terms integrated correctly

(So if μ = 0 could score M0M1 here)

A1 Correct answer $4x-4(x\ln x-x)+x(\ln x)^2-2x\ln x+2x(+c)$ with or without *c*. Accept unsimplified and isw.

ddM1 Attempts to substitute both correct limits into the result of their integral. Both previous M's must have been scored. If there has been small slips simplifying the result of the integral before use of limits then allow M1

A1 Correct solution only $\pi\mathrm{e}(2\mathrm{e}-5)$

Special case: For those who misunderstand/ misread and think that $y=(\ln x)^2$ so Volume = $\int\pi y^2\mathrm{d}x=\int\pi(\ln x)^4\,\mathrm{d}x$ - the first B1 may be exceptionally awarded. If anyone appears to make progress with a method for this integration (not rubbish) please send to review.

This gains 1/6 marks

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom